Analysis of a CPW on Electric and Magnetic Biaxial Substrate

Geneviève Mazé-Merceur, Smail Tedjini, and Jean-Louis Bonnefoy

Abstract— **This paper deals with a new formulation of the Spectral Domain Technique (SDT) for the analysis of the general case of uniaxial/biaxial, electric/magnetic anisotropic multilayer planar structures. As an illustration of the capabilities of this formulation we apply it to the analysis of shielded CPW structures. The dispersive properties of the fundamental and higher order modes in various cases of electric/magnetic anisotropy as well as the induction electric and magnetic lines are calculated. Such results, and particularly the induction lines, may be used to predict the behaviour of the studied structure as well as to point out its sensitive geometrical and electrical parameters. Some general rules will be discussed which lead to a better understanding of the effect of anisotropy.**

I. Introduction

Anisotropic materials are of great interest in microwave and millimeter wave applications [1]. They include gyroelectric and gyromagnetic materials, sapphires, certain ceramics, pertinent substrates for high speed opto-electronic applications (i.e., $LiNbO_3$), but also microwave absorbers such as planar hexaferrites or composites loaded with fibers [2].

In most cases, when analyzing integrated circuits, the anisotropy is neglected and the materials are assumed to be isotropic. This could be an acceptable approximation for quasi-TEM solutions (i.e., microstrip topology) and slightly anisotropic material, but for high frequencies and/or non TEM configurations the later approximations do not lead to reliable results and rigourous dynamic techniques must be applied in order to get an accurate solution. However, the existing results are mostly related to microstrip configuration with special forms of permittivity and permeability tensors [3], [4]. Only little has been done concerning full wave analysis of non TEM configurations [5]. As far as we know, a generalized hybrid modes analysis of planar structures including uniaxial/biaxial magnetic/dielectric layers, in arbitrary number, has not been given yet except by E. Drake *et al.* [6]; but no results are found including consideration of induction field lines.

The intention of this paper is to generalize the Spectral Domain Technique [7] so that it encompasses all the shielded planar configurations including biaxial substrates. In order to get better understanding of physical effects of the anisotropy on the electromagnetic propagation, the electric and magnetic induction field pattern will be carried out.

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II. Propagation in Anisotropic Materials

In the following sections, we consider biaxial media for which the relative permittivity (ε_r) and permeability (μ_r) tensors are represented by diagonal matrices in a rectangular cartesian reference system. So, the most general form of these tensors takes the form:

$$[\varepsilon_r] = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \qquad [\mu_r] = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \tag{1}$$

The ε_i and $\mu_i\,(i = x, y, z)$ elements are dimensionless and are, in the general case, complex and frequency-dependent. Let us consider the propagation following the Oz direction (see Fig. 2) in an homogeneous biaxial anisotropic material. Substituting the permittivity and the permeability tensors given in (1) into Maxwell equations and using the classical complete bases of TE, TM or LSE, LSM [9] modes, it can be shown that the general solution is a superposition of:

1. LSE($E_x = 0$), LSM($H_x = 0$) modes only if $\varepsilon_z \cdot \mu_y = \varepsilon_y \cdot \mu_z$,
2. LSE($E_y = 0$), LSM($H_y = 0$) modes only if $\varepsilon_z \cdot \mu_x = \varepsilon_x \cdot \mu_z$,
3. TE($E_z = 0$), TM($H_z = 0$) modes only if $\varepsilon_x \cdot \mu_y = \varepsilon_y \cdot \mu_x$,
4. Hybrid modes in the other cases

where $E_i, H_i, (i = x, y, z)$ are the electric and magnetic fields respectively.

Since the considered medium is infinite and homogeneous, the cases 1) and 2) are physically one and the same. We can go from case 1) to case 2) by a $\pi/2$ rotation of the coordinate system about the z-axis. However, in planar configurations including biaxial anisotropic layers both later cases 1) and 2) must be considered.

III. Analysis

The most general planar structure is a shielded planar structure. A particular case is shown Fig. 2. Its walls are considered to be perfect conductors. Only one interface (plane $x = 0$) is metallized (several strips or slots). The number of layers is arbitrary (N layers $x < 0$ and K layers $x > 0$), the media filling the waveguide are either isotropic, or anisotropic with tensors of the form given in Section II.

l_i is the thickness of ith layer, $\varepsilon_r(i)\,(\varepsilon_x(i), \varepsilon_y(i), \varepsilon_z(i))$ is its complex permittivity, and $\mu_r(i), (\mu_x(i), \mu_y(i), \mu_z(i))$ its complex permeability.

The Spectral Domain Technique (SDT) is well suited to analyse planar structures. It is mainly a Fourier-transformed version of the integral equation method applied to multilayer printed lines. The electromagnetic field components are expressed either in terms of a continuous or a discrete Fourier spectrum depending on whether the structure is open or closed.

LSE and LSM modes are chosen in an isotropic case, because these modes are coupled only on the metallized interface which simplifies the analytical development leading to recurrent expressions and a high degree of flexibility in manipulating multilayer structures [9].

When the permittivity ε and the permeability μ are diagonal tensors, and mode expansion basis must be chosen carefully following the conclusions given in Section II. A transfer matrix is established for each layer, expressing the tangential fields at one face of the layer as a function of the tangential fields at the other face. Hence this matrix depends on whether constitutive relations exist between the terms of the tensors or not.

Application of the boundary conditions at the interfaces along with a chain matrix formalism leads to the following relation between the Fourier components of tangential current densities: $J_{y,z}(\alpha)$ and tangential electric fields: $E_{y,z}(\alpha)$ at the $x = 0$ plane.

$$\begin{bmatrix} I_z(\alpha) \\ I_y(\alpha) \end{bmatrix} = [G] \begin{bmatrix} E_z(\alpha) \\ E_y(\alpha) \end{bmatrix}$$

G is a second rank matrix, known as the Green's Dyadic and takes an analytical form.

The Galerkin Method is applied to solve this equation. Basis functions are to be used. The choice of a particular basis is linked to the convergence of results and is a classical problem is the Spectral Domain Techniques. In this study we choose simple trigonometric functions as a basis because it is the simplest to manipulate with regards to the Fourier transforms and thus leads to a significant CPU time saving leading to linear system of equations which gives the propagation constant. Thus, the impedance and electromagnetic field in the structure are straightway obtained.

IV. Application

Results obtained based upon this formulation have been compared to previous published results. As an example, we have reported on Fig. 1 the values of the effective dielectric constant $\varepsilon_{\text{eff}} = (\kappa/\kappa_0)^2$ (where κ is the phase constant in the structure, and κ_0 is the phase constant in the free space) obtained by our formulation for a bilateral finline on biaxial substrate and those obtained by H.-Y. Yang *et al.* [10]. They are in good agreement (the error is less than 3%). Moreover, some of the presented numerical results have been confirmed experimentally [11].

As an application of this procedure, we will consider in the next a shielded CoPlanar Waveguide (CPW). It enables to propagate two modes: quasi-TEM and quasi-TE. The effect of electric/magnetic anisotropy will be investigated for both modes. To this end, we will calculate the value of $\varepsilon_{\text{eff}} * \mu_{\text{eff}}$, defined as:

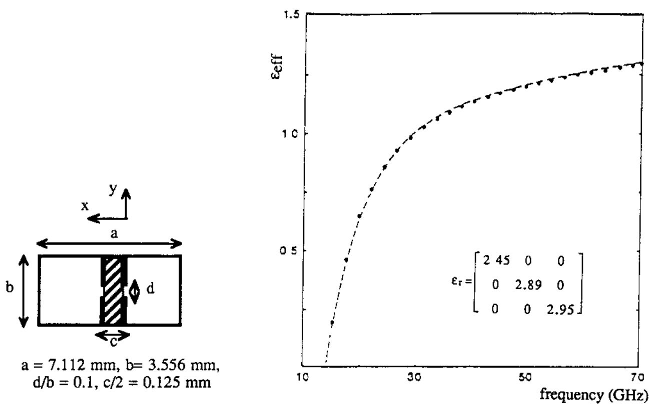


Fig. 1. ε_{eff} versus frequency for a bilateral finline on biaxial substrate - - - - - results of Yang and Alexopoulos, • • • • • our results.

$$\varepsilon_{\text{eff}} * \mu_{\text{eff}} = \left(\frac{\kappa}{\kappa_0}\right)^2$$

as well as the electric and magnetic induction field lines in the cross section of the CPW. The typical configuration of this structure is shown in Fig. 2. The shielding waveguide is WR28: $a = 7.112$ mm, $b = 3.556$ mm.

First, the case: $\mu_r = 1, \varepsilon_r$ anisotropic will be considered, and the effect of anisotropy will be studied, after that the case ε_r isotropic, μ_r anisotropic will be considered, and finally the case where ε_r and μ_r are both anisotropic will be analyzed.

Electric Anisotropy: $\mu_r = 1, \varepsilon_r = (15, 10, 15)$ *(Since* $\mu_r = 1, \varepsilon_{\text{eff}} * \mu_{\text{eff}} = \varepsilon_{\text{eff}}$*):*

The $(\varepsilon_{\text{eff}})$, as a function of the frequency, of the dominant (quasi-TEM) and the first higher (quasi-TE) modes are shown on Fig. 3. These values are compared with the ones obtained when assuming the substrate isotropic with $\varepsilon_r = 10$ or $\varepsilon_r = 15$. One notices that, for the quasi-TE mode, the anisotropic structure behaves as if it were isotropic, with $\varepsilon_r = 10$. This is due to the overall orientation of the induction electric field (D_t) mainly along Oy, shown on Fig. 4(b) at $f = 34.8$ GHz. Moreover, ε_{eff} increases with the frequency because the D_t field is more and more confined near the metallization plane. On the other hand, no preferential orientation exists for the quasi-TEM solution as shown on Fig. 4(a). Ones notices this mode has an electric wall in the center of the guide where the first higher mode has a magnetic wall. As a conclusion, the substrate may be assumed isotropic for the first higher mode but no general rule can be given for the quasi-TEM mode.

Physically, the effect of the anisotropy in the case $\varepsilon_r = (10, 15, 15)$ can be straightway deduced from the above results: we can also predict that the structure can not be assumed as an isotropic one for the dominant mode. On the contrary, as the field pattern of the first higher mode exhibit a preferential direction along Oy, the structure will behave as if it were isotropic with $\varepsilon_r = 15$. Moreover, as the E_z component of the electric field of both modes is small, compared to the E_x and E_y components, the anisotropic structure with a substrate

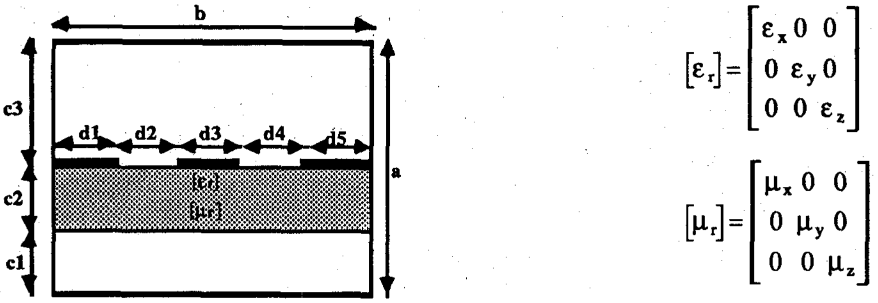


Fig. 2. Coplanar waveguide, $c_1/a = 0.4; c_2/a = 0.1; c_3/a = 0.5; d_1 = d_2 = d_3 = d_4 = d_5 = b/5$.

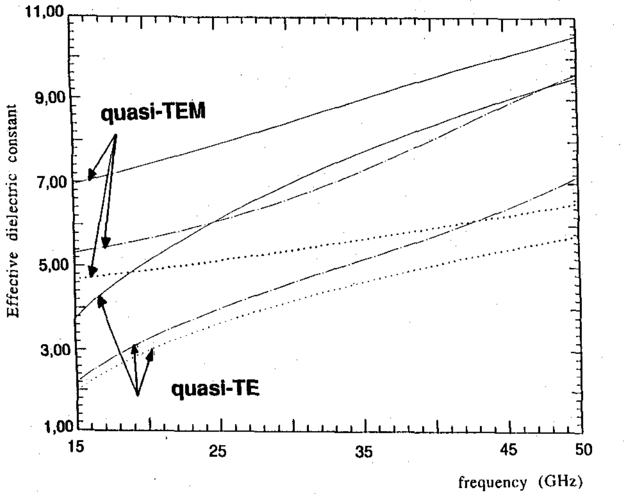
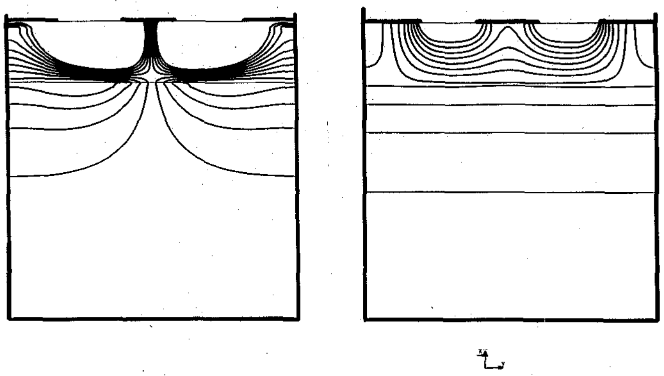
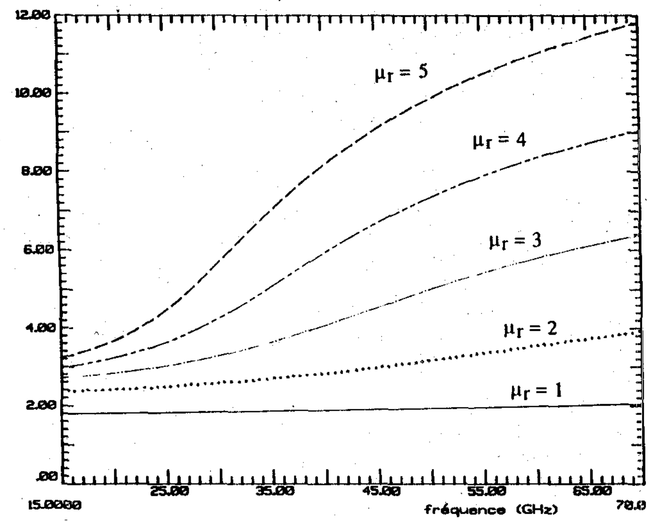


Fig. 3. Effective dielectric constant versus frequency (GHz) $\varepsilon_r = 10$: · · · · · ·, $\varepsilon_r = 15$: ___, $\varepsilon_r = (15, 10, 15)$: _ · _ · _.

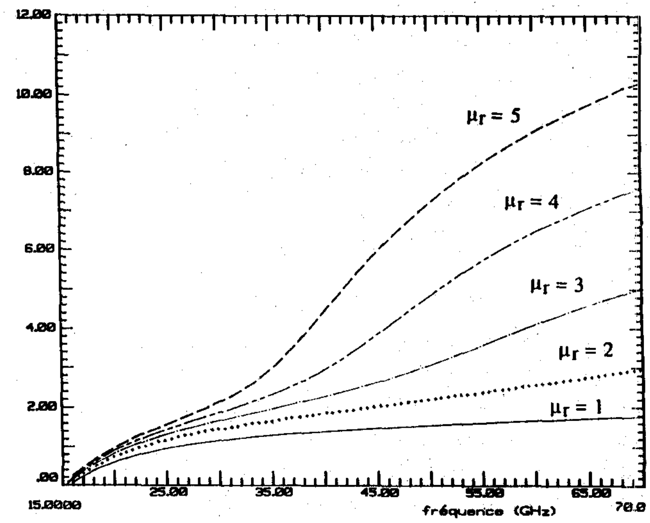


(a) (b)

Fig. 4. D_t field lines in half the cross section of the CPW, $x \uparrow_{\rightarrow y}$ $\mu_r = 1, \varepsilon_r = (15, 10, 15), f = 34.8$ GHz, (a) Dominant mode: $\kappa = 1952$ rad/m. (b) First higher mode: $\kappa = 1659$ rad/m.



(a)



(b)

Fig. 5. $\varepsilon_{\text{eff}} \cdot \mu_{\text{eff}}$ versus frequency (GHz), $\varepsilon_r = 3$, μ_r varying: (a) Dominant mode. (b) First higher mode.

of permittivity $\varepsilon_r = (15, 15, 10)$ will behave as an isotropic one, with $\varepsilon_r = 15$.

These predictions have been confirmed by the theoretical results.

Magnetic Isotropy ($\varepsilon_r = 3$):

For better understanding of the magnetic anisotropy effect in the studied structure, let us consider the isotropic case for a $\varepsilon_r = 3$ and several values of the isotropic permeability ($\mu_r = 1, 2, 3, 4, 5$). The variations of $\varepsilon_{\text{eff}} \cdot \mu_{\text{eff}}$ versus frequency are given on Fig. 5(a) and (b) for the fundamental and first higher mode. It appears that the dispersion increases significantly with μ_r.

The B_t and D_t field lines relative to magnetic and non-magnetic substrate are shown on Fig. 6 for the fundamental mode. As far as μ_r increases, we observe a strong concentration of B_t and D_t in the substrate and a preferential orientation of B_t along Oy and of D_t along Ox. This phenomenon may

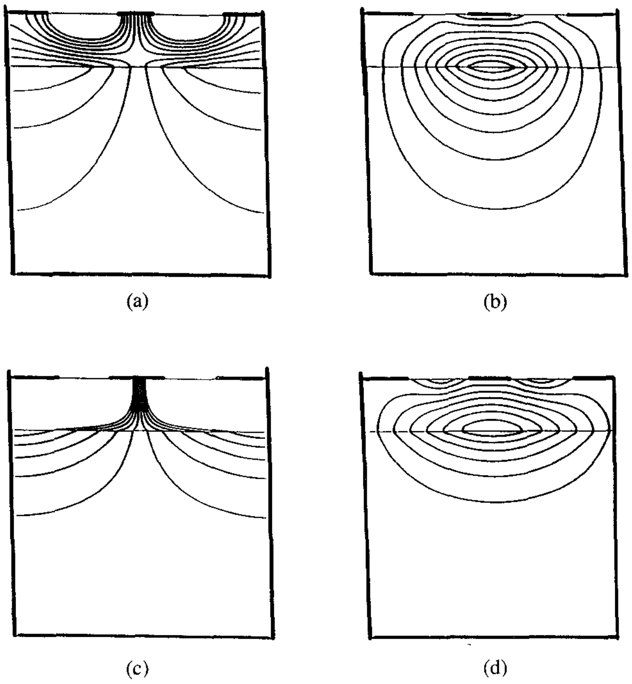


Fig. 6. D_t (a), (c) and B_t (b), (d) field lines in half the cross section of the CPW $^x\uparrow_{\_\_\to y}$ $\varepsilon_r = 3$. $f = 34.8$ GHz, dominant mode: $\beta = 996$ rad/m. (a), (b): $\mu_r = 1$, (c), (d): $\mu_r = 5$.

be understood if we remember that Ex and Hy components have the same behavior with respect to Maxwell equations.

Magnetic Anisotropy ($\varepsilon_r = 3$):

Let us consider the effect of magnetic anisotropy. For the three different cases of magnetic anisotropy the variations of $\varepsilon_{\text{eff}} \cdot \mu_{\text{eff}}$ versus frequency are given Fig. 7(a) and (b) for the dominant and the first higher modes. On the same figure we reported the two isotropic cases ($\mu_r = 1, 5$). Only anisotropy along Oy introduces strong influence on the dominant mode compared to the case $\mu_r = 1$, and is quite enough sensitive to the magnetic anisotropy for the first higher mode.

Electric and Magnetic Anisotropy

Finally we will consider the more general case with electric and magnetic anisotropy. Let $\varepsilon_r = (5, 3, 3)$ and $\mu_r = (1, 5, 1)$. The variations of $\varepsilon_{\text{eff}} \cdot \mu_{\text{eff}}$ for the fundamental mode are given Fig. 8, and compared to isotropic cases. The results show a quasi TEM behavior when the substrate is isotropic ($\varepsilon_r = 3, \mu_r = 1$ or 5). As far as the substrate becomes anisotropic a strong dispersion is observed which increases significantly with the frequency, the permeability and the permittivity of the substrate. Furthermore the values of $\varepsilon_{\text{eff}} \cdot \mu_{\text{eff}}$, at low frequency, are higher in isotropic case ($\varepsilon_r = 3, \mu_r = 5$) than in anisotropic one. Physically, this is the expected result because the electromagnetic field in mainly transverse at low frequency and then Ox and Oy axes are the sensitive directions. Now the "equivalent" $\varepsilon_r \cdot \mu_r$ in the cross section is higher in isotropic case. On the contrary, for high frequency, the mode becomes so hybrid leading to more and more dispersion which increases with anisotropy as predicted before so that anisotropic structure becomes the more dispersive one.

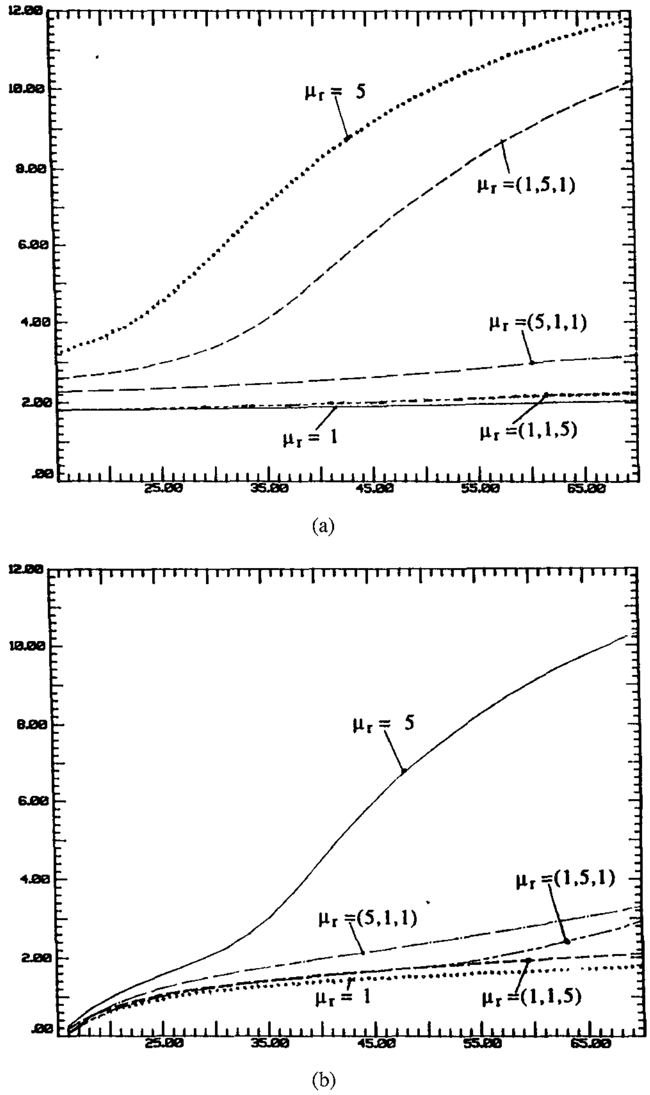


Fig. 7. $\varepsilon_{\text{eff}} \cdot \mu_{\text{eff}}$ versus frequency (GHz), anisotropy of μ_r varying $\varepsilon_r = 3$. (a) Dominant mode. (b) First higher mode.

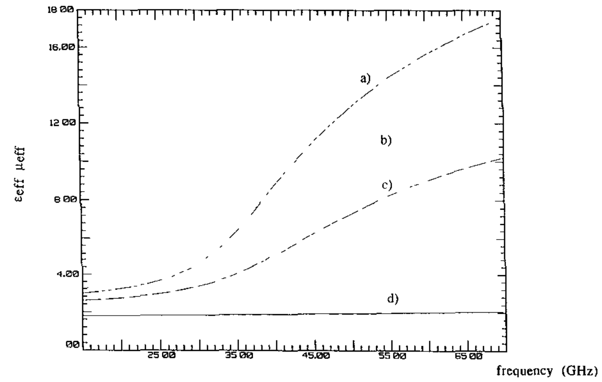


Fig. 8. $\varepsilon_{\text{eff}} \cdot \mu_{\text{eff}}$ versus frequency (GHz), dominant mode: (a) $\varepsilon_r = (5, 3, 3), \mu_r = (1, 5, 1)$. (b) $\varepsilon_r = 3, \mu_r = 5$. (c) $\varepsilon_r = 3, \mu_r = (1, 5, 1)$. (d) $\varepsilon_r = 3, \mu_r = 1$.

V. Conclusion

In order to study the effect of anisotropy on the properties of planar structures a suitable formulation of the Spectral Domain Technique (SDT) is applied. It enables to study the general case of biaxial, electric and/or magnetic and anisotropic circuits, with arbitrary number of layers. Usually the SDT is used to calculate the global parameters of a given structure (mainly, propagation constant and characteristic impedance). In this actual paper, we have also extended it in order to calculate the electromagnetic field at any point of a cross section. As illustration of the capabilities of this formulation, we applied it to a shielded CPW. The obtained results, and particularly the induction lines, are of great interest because they enable to predict the behavior of the studied structure as well as to point out the sensitive geometrical and electrical parameters of the structure. Some physical in sight is given that leads to a better understanding of the effect of anisotropy and could be very useful when dealing with inverse problems. The theoretical predictions have been confirmed by the numerical results.

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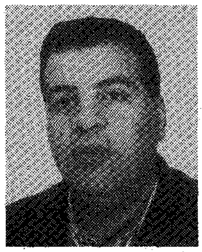

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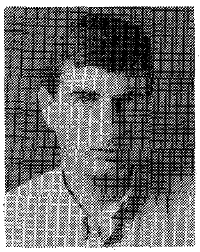

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